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NEW PROOFS OF TRIANGLE INEQUALITIES

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Dedicated to Prof. A.K. Agarwal on his 70^{th} ***Birth Anniversary***

Abstract: We give three new proofs of the triangle inequality in Euclidean Geometry. There seems to be only one known proof at the moment. It is due to properties of triangles, but our proofs are due to circles or ellipses. We aim to prove the triangle inequality as simple as possible without using properties of triangles. In other words, we suggest proofs without using paper and pen.

Keyword and Phrases: Triangle inequality, circle, ellipse, focci of ellipse, Euclidean geometry, algebraic geometry.

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1. Introduction

We consider triangle inequalities for triangles on Euclidean space. For convenience, let the dimension of the space be 2. We write $\triangle \mathrm{ABC}$ for the triangle with three vertices $\mathrm{A}, \mathrm{B}, \mathrm{C} \in \mathbb{R}^2$. Hereafter, AB denotes the segment from a point A to a point B, and $\angle \mathrm{ABC}$ the angle made by segments AB, BC. Moreover, $\overline{\mathrm{PQ}}$ stands for the length of the segment PQ in $\mathbb{R}^2$.

The triangle inequality asserts that the sum of any two sides of a triangle is *strictly* bigger than the remaining third side. This geometric inequality is well known as one of the most fundamental and classical theorems in Euclidean geometry: